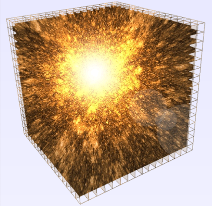

Real time lattice simulations *and* heavy quarkonia beyond deconfinement

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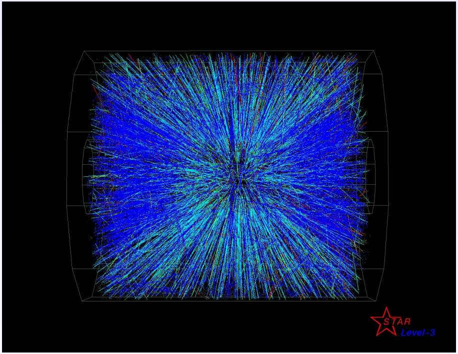


Heavy quarkonia beyond deconfinement?

Charmonium and Bottomonium

Heavy Quarkonia

- Bound systems consisting of a heavy quark and the corresponding antiquark:

  $c\bar{c}$ (Charmonium) and $b\bar{b}$ (Bottomonium)

Discovery and relevance for $T = 0$ physics

- The first $c\bar{c}$ state, the J/ψ, was found 1974 by B. Richter and S. Ting and led to the discovery of the charm quark which was predicted by earlier QCD calculations.
- An investigation of the energy spectrum of charmonium resonances from e^+e^--annihilation played an important role in establishing QCD as the correct theory of strong interactions.

The potential of the strong interactions

Description via a Schrödinger equation

Due to the high mass of the constituents ($m_q \gg E_{kin}$) heavy quarkonia can be described by a nonrelativistic Schrödinger equation:

$$i\partial_t \psi = (-\frac{\triangle_i}{2\mu} + \hat{V})\psi$$

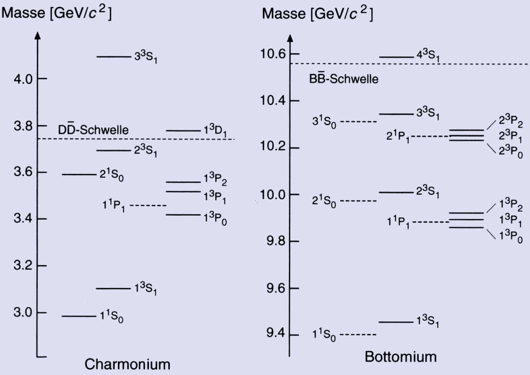
Phenomenological potential

$$\hat{V} = k\hat{r}^{\,1)} - \frac{4}{3}\frac{\alpha_s(\hat{r})}{\hat{r}}^{\,2)}$$

1. Linear part: Due to the formation of a flux tube (predicted by strong coupling lattice QCD, k ≈ 1GeV/fm for $c\bar{c}$)
2. Coulomb part: similar to QED (predicted by perturbation theory, $\alpha_s \approx 0.15 - 0.25$)

Level diagram

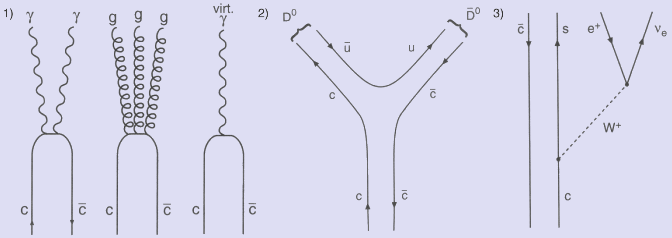
The level diagrams of charmonium and Bottomonium



Solid (dotted) lines indicate (un)discovered states predicted by the Schrödinger equation.

Decay channels at T=0

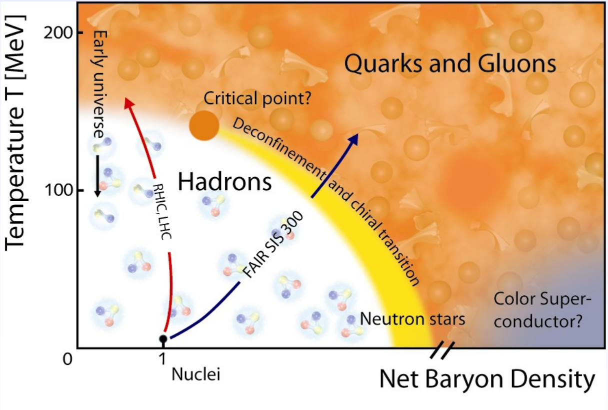
Decay channels



Description

1. Annihilation to photons or gluons (relevant at low energy)
2. String breaking (relevant at high energy)
3. Electroweak decay (suppressed)

Beyond deconfinement ...



(Assumed phase diagram of the strong interactions)

Debye screening

$$\hat{V} = k\hat{r} - \frac{4}{3}\frac{\alpha_s(\hat{r})}{\hat{r}}$$

Modification of the potential at $T > T_c$

1. Flux tubes are assumed to break down at $T > T_c$ (*for now*).
2. A modification of the Coulomb part of the static potential is to be expected due to the thermal gluon mass $\Pi(p^0, \mathbf{p})$.

The existence of bound states beyond deconfinement depends on the exact form of the modified Coulomb potential.

Debye screening

Quick reminder: Linear response theory.

The modification $\delta\hat{A}(t)$ of the expectation value of an operator $\hat{A}$ in the presence of a weak disturbance $\epsilon\hat{V}(t)$ is to first order in ϵ:

$$\delta\hat{A}(t) = -i\int_{t_0}^{\infty} dt'\Theta(t-t') < [\hat{A}, \epsilon\hat{V}(t')] >$$

Derivation of the modified potential (I)

- Assume a quark-gluon plasma in thermal equilibrium which is disturbed by a weak color current j^a_μ:

$$\hat{V}(t) = \int d^3x j^a_\mu(x)\hat{A}^a_\mu$$

- The expectation value A^a_μ of the gauge field is given by:

$$A^a_\mu = \delta\hat{A}^a_\mu$$

Debye screening

Derivation of the modified potential (II)

- From linear response theory we find:

$$A^a_\mu(x) = -i \int d^4x' \Theta(t-t') < [\hat{A}^a_\mu(x), \hat{A}^b_\nu(x')] > j^{\nu(b)}(x')$$

This translates to the following fourier space expression:

$$A^a_\mu(q) = -iD^{R(ab)}_{\mu\nu}(q) j^{\nu(b)}(q)$$

- Inserting the gluon propagator in Landau gauge we arrive at:

$$A^a_0(q) = -\frac{\rho^a(q)}{q^2 - \Pi}, \quad A^a_i(q) = -\left(\frac{P^L_{ij}}{q^2 - \Pi} + \frac{P^T_{ij}}{q^2 - G}\right) j^{j(a)}(q)$$

Debye screening

Derivation of the modified potential (III)

- Concentrating on a point charge with $\rho^a(x) = Q^a \delta(\mathbf{x})$ we find the following color electric field:

$$E_i^a(q) = F_{0i}^a = -\frac{i q_i Q^a}{\mathbf{q}^2 - m_D}$$

- Returning to Minkowski space the (gauge invariant) potential can now simply be extracted via

$$E_i^a(\mathbf{x}) = -\partial_i V^a(\mathbf{x})$$

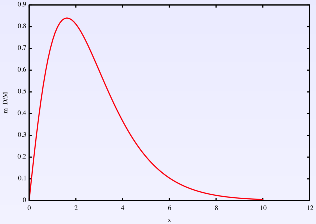
The Debye screened potential at $T > T_c$:

We thus arrive at the following Yukawa-type potential:

$$V^a(r) = \frac{Q^a}{r} e^{-m_D r}$$

(This potential evidently allows bound states!)

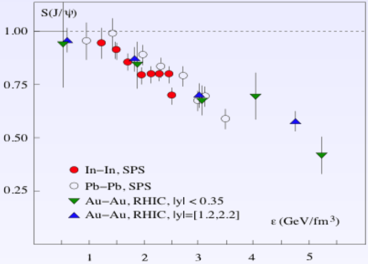
Bound states beyond T_c



A simple argument on bound states ... (Matsui, Satz, 1986)

- Minimizing the shown estimate for the energy of a bound state
$$E(r) = 2M + \frac{1}{2Mr^2} + V(r)$$
the following relation for the ground state is found:
$$x(x+1)e^{-x} = \frac{m_D}{M} \text{ with } x = m_D r \text{ (plotted above)}$$
- It follows that for $m_D < 0.84M$ bound states are possible.

Experimental observations



J/ψ suppression in heavy ion collisions

Color screening effects on the J/ψ or higher charmonium excitations[Karsch et al.,2005] may have been observed at RHIC but J/ψ-suppression is expected by some hadronic models as well...[Kapusta]

Spectral function and lattice studies

Definition: The spectral function

The spectral function $\rho(q^0)$ is defined by the relations

$$\rho(q^0) = \frac{1}{2}\left(1 - e^{-\beta q^0}\right)\tilde{C}_>(q^0), \quad C_>(q^0) = \int_{-\infty}^{\infty} dt \int d^3x\, e^{iQx}\, \langle J^\mu(x) J_\mu(0)\rangle$$

using the mesonic correlator $\tilde{C}_>(q^0)$ and the current $J^\mu = \bar{\psi}\gamma^\mu\psi$.
The dilepton production rate is proportional to the spectral function.

Results from Bayesian lattice simulations Petreszky,2006

The spectral function was extracted in several recent lattice studies using MEM techniques. The following results were found:

- *Charmonium:* $1S$ states are expected to survive deconfinement until $T \sim 1.5T_c$, while $1P$ states dissolve already at $T \sim 1.1T_c$
- *Bottomonium:* $1P$ states dissolve at $T \sim 1.5T_c$, while $1S$ states are even expected to expected to remain stable for $T \gtrsim 2T_c$

The real-time static potential

$$\left[i\partial_t - \left(2M - \frac{\triangle_r}{M} \right) - V \right] C_>(t,r) = 0$$

(Assumed Schrödinger equation of the current correlator)

Definition of a quarkonium potential (Laine et al., 2006)

To overcome difficulties faced by potential models at $T > T_c$ a dynamical definition of the potential based on the mesonic correlator

$$C_>(t,r) = \int d^3\mathbf{x} < \bar{\psi}(t, \mathbf{x} + \mathbf{r})\gamma^\mu W_{\mathbf{r}} \psi(t, \mathbf{x}) \bar{\psi}(0,0)\gamma_\mu W_{\mathbf{-r}} \psi(0, \mathbf{r}) >$$

was suggested. The *real time static potential* is defined in the limit $M \to \infty$ where the correlator reduces to a Wilson loop.

The real-time static potential

$$[i\partial_t - {\color{red}V}(t,r)]\, C_>(t,r) = 0$$

(The static potential)

Calculation in perturbation theory

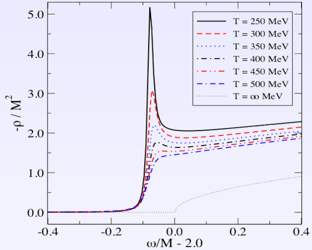
A perturbative calculation of the static Wilson loop yields the result

$$\lim_{t\to\infty} V(t,r) = -\frac{g^2 C_F}{4\Pi}[m_D + \frac{exp(-m_D r)}{r}] - i\frac{g^2 T C_F}{2\pi}\phi(m_D r)$$

with $\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[1 - \frac{sin(zx)}{zx}\right]$

with the real part being the usual thermal potential and an additional *imaginary part* originating from Landau damping.

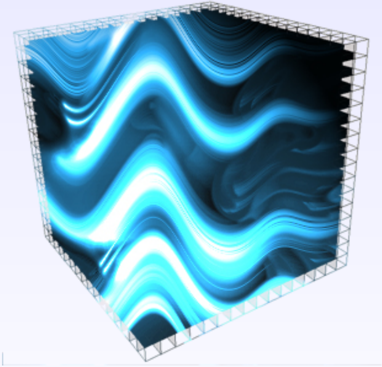
The imaginary part



Role of the imaginary part

The potential has been reinserted into the Schrödinger equation and the spectral function calculated.[Laine,2007]
A finite resonance width has been introduced to the potential model.
The spectral function is in agreement with recent potential model results based on Bayesian lattice calculations![Petreczky,2007]

...but how can a dynamically defined quantity be investigated on the lattice ?



Real time lattice simulations

Introduction

Real time lattice simulations are usually the attempt to study the long range dynamics of a quantum field theory in a semiclassical context.

In the following an appropriate framework for a semiclassical treatment of the quark gluon plasma including leading order quantum effects will be introduced.

Lets start with statistics...

Dimensional reduction

$$Z = N \int_{\phi_i(\beta)=\phi_i(0)} D[\phi_i]\, exp\left[-\int_0^\beta d\tau \int d^3x \mathcal{H}(x,\tau)\right]$$

(A generic thermal partition function)

Dimensional reduction

In the high temperature limit $\beta \to 0$ the integration over euclidean time can be executed immidiately, yielding the following canonical partition function:

$$Z \simeq N \int D[\phi_i]\, exp\left[-\beta \int d^3x \mathcal{H}(x,0)\right]$$

see for example: Blaizot, Iancu: The Quark-Gluon Plasma, 2001

Application to the Yang-Mills theory

$$Z = \int \mathcal{D}[A, B, W]\, e^{-\beta H} \quad \text{with}$$

$$H(x) = \frac{1}{2} \int d^3x \left(E^a E^a + B^a B^a + m_D^2 \int \frac{d\Omega}{4\pi} W^a(x, v) W^a(x, v) \right)$$

(Partition function of the Hard-Thermal-Loop effective theory)

The Hard-Thermal-Loop effective theory Blaizot,Iancu,Nair,1993

The dimensional reduction scheme was applied to the Yang-Mills theory at finite temperature by integrating out the UV degrees of freedom beyond a scale $\mu \ll T$ followed by a dimensional reduction of the partition function.
The effective field $W(\mathbf{x}, \mathbf{v})$ describes the charge density of hard modes at $\mathbf{x}$ moving in the direction $\mathbf{v}$.

Claim: Dynamical observables can be calculated by evolving the ensemble using the classical equations of motion...

Gauge field dynamics

The original claim (Grigoriev, Rubakov, 1988)

Quantum fields can be described using classical dynamics provided the elementary excitations obey classical statistics.

Dynamical scales of the Yang-Mills theory (Bödecker et al., 1995)

A closer look at the classical limit using real time perturbation theory reveals the following relevant scales for the Yang-Mills theory:

- $\mathbf{k \sim T}$ (Characteristic plasma scale)
  Characteristic scale of the hard excitations.
- $\mathbf{g^2 T < k < gT}$ (Collective dynamics)
  A classical approximation of the gauge field is possible.
  Hard-thermal loop interactions should be taken into account.
- $\mathbf{k < g^2 T}$ (Nonperturbative dynamics)
  Transition to the strong coupling regime.

Equations of motion

$$D_\mu F^{\mu\nu} = j^\mu \qquad \textit{(1. Yang-Mills equation)}$$

$$v^\nu D_\nu W(x, \mathbf{v}) = F^{0\nu}(x) v_\nu \qquad \textit{(2. HTL equation of motion)}$$

$$j^\mu = m_D^2 \int \frac{d\Omega_\mathbf{v}}{4\pi} v^\mu W(x, \mathbf{v}) \qquad \textit{(Color current } (v^0 = 1)\textit{)}$$

(Effective theory equations of motion)

The classical equations of motion

The dynamics of the effective Hard-Thermal-Loop theory can be described using the shown Hamiltonian equations of motion.

These equations can now be used to evolve the statistical ensemble of 3-dimensional field configurations. A numerical solution requires an appropriate discretization scheme...

Hamiltonian lattice simulations

The lattice setup (Kogut, Susskind, 1974)

In the following we will adopt a formalism where the spatial degrees of freedom are discretized on a three dimensional lattice while the (Minkowski-) time coordinate remains continouus. This formalism was originally introduced to study flux tube dynamics on the lattice.

Gauge fields in the Hamiltonian formalism

- The temporal gauge $A_0 = 0$ is chosen.
- Spatial gauge fields are discretized as usual as the parallel transport connecting neighbouring lattice points.
- The colour electric field is defined from the dynamics of the spatial links:

$$\dot{U}_i(x) = iE_i(x)U_i(x)$$

The classical Yang-Mills field on the lattice

$$S = \frac{1}{2T} \int d^4x Tr F_{\mu\nu} F^{\mu\nu}$$

(The Yang-Mills action)

Wilsons action for real-time lattice simulations

$$S = \beta_3 \int dt \sum_{\mathbf{x}} \left(\frac{1}{2N} Tr \dot{U}_i^+ \dot{U}_i{}^{1)} - \sum_{i<j} \left[1 - \frac{1}{N} Re Tr U_{ij} \right]^{2)} \right)$$

1. Electric part: Found by discretizing the temporal part of the action
$$S_E = \frac{1}{T} \int d^4x Tr E_i E^i, \quad (E_i = F_{0i})$$
using the definition of the electric field on the lattice.
2. Magnetic part: The remaining spatial part of the action is discretized in analogy to the 4-dimensional Wilson action.

The space of physical states

$$L(GU_i) = L(U_i)$$

(Local gauge invariance)

Physical states

A physical state can be defined by invariance of the lagrangean L under arbitrary local gauge transformations G.

Gauss constraint

A local gauge transformation $G(x)$ at a lattice site x effects all emanating links. The following constraint is derived using the real time action:

$$\sum_i \left[E_i(x) - U_{-i}(x) E_i(x-i) U^{+}_{-i}(x) \right] = 0$$

Gauss's law is recovered in the continuum limit.

Note: In contrast to euclidean lattice simulatons great care has to be taken to ensure the gauge invariance of lattice states.

Equations of motion

Derivation of the equations of motion for the electric field

The classical lattice equations of motion for the electric field can be derived by varying the action with respect to the link variables.

$$\delta S = 0$$

The gauge field evolution is in turn defined by the electric field.

Equations of motion (Ambjorn, Askgaard et al., 1991)

The complete equations of motion:

$$\dot{U}_i(x) = iE_i(x)U_i(x) \qquad \text{(defines the electric field)}$$

$$\dot{E}_i^a = 2\sum_{|j|\neq i} ImTr(T^a U_{ij}) \qquad \text{(see above)}$$

(The Gauss constraint remains satisfied, Scaling: $a^2 g E_i^a \rightarrow E_i^a$, $a^{-1}t \rightarrow t$)

The statistical ensemble

Generation of the ensemble (Moore,1996)

A statistical set of starting configurations respecting gauss's law has to be created:

1. Pregenerate the gauge fields by a three dimensional Monte Carlo (Moore: U=1).
2. Draw the electric fields from a gaussian distribution.
3. Project on the space of physical configurations using:

$$E_i(x) \rightarrow E_i(x) + \gamma(U_i C(x+\hat{i}))U_i^+ - C(x))$$

(C: Violation of gauss's law)

4. Evolve the fields using the EOM and repeat from (2) until the gauge fields have completely thermalized.

A conceptual problem

$$E(T) = \frac{T}{6\pi^3}\mu^3$$

(Energy density of the classical theory)

UV divergencies of the classical theory

UV-divergencies similar to those in classical electrodynamics that led to the invention of quantum mechanics are encountered in the classical Yang-Mills theory.

To avoid these divergencies the full effective Hard-Thermal-Loop theory including leading order quantum effects from hard thermal modes can be implemented. (Bödecker,Moore et al.,1999)

Hard thermal loop simulations

$$\dot{W}_n(x) = v_n^i(2\bar{E}_i(x) - [P_i W_n(x+i) - P_{-i} W_n(x-i)]) \qquad \textit{(EOM: HTL)}$$

$$j^i = (x)\frac{(am_D)^2}{N_p} v_n^i W_n(x) \qquad \textit{(Color current)}$$

(Lattice equations of motion)

"Discoball" discretization (Romatschke, 2007)

To simulate the full effective theory the missing HTL equation of motion can be implemented using the above scheme, where the sphere of directions has been discretized using platonian solids. This equation is then coupled to the lattice Yang-Mills field via the current.

Used definitions:

Averaged electric field: $\bar{E}_i(x) = \frac{1}{2}(E_i(x) + P_{-i}E_i(x-\hat{i}))$
Parallel transport: $P_i\phi(x+\hat{i}) = U_i(x)\phi(x+\hat{i})U_i^+(x)$, Discoball vertex: v_n

Benefits and shortcomings

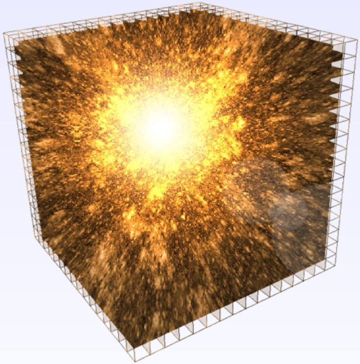
The benefits and shortcomings of real time simulations

The benefits of real time lattice simulations include:

- (Semi-) classical simulations are an effective way to study the long range dynamics of the quark gluon plasma.
- The effective theory approach makes the relevant physics accessible.

Real time simulations are however plagued by several problems:

- For correlators sensitive to quantum corrections the continuum limit is often ill defined.
- The long term behaviour of many correlators is dominated by hydrodynamic fluctuations which are not correctly reproduced on the lattice.(Arnold,Yaffe,1997)
- Care has to be taken to ensure the gauge invariance of lattice states.



Some results...

Extraction of the real time static potential

$$(i\Delta_t - V(t,r))\, C_>(t,r) = 0$$

(The real time static potential)

Extraction from Wilson loop dynamics

- A Wilson loop of extent (r,t) was averaged over an ensemble of configurations using classical or HTL improved simulations:

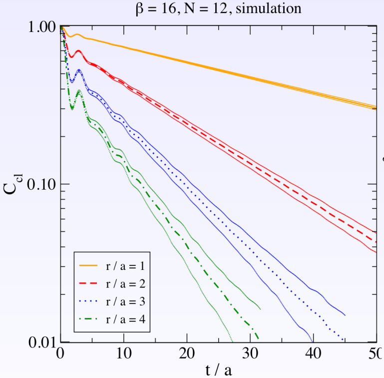
$$C_>(t,r) = \frac{1}{N} \left\langle Tr\left(W^+(t,x)W(0,x)\right)\right\rangle_{|x|=r}$$

- The potential was extracted using the 3-point derivative Δ_t:

$$V(t,r) = i\frac{\Delta_t C_>(t,r)}{C_>(t,r)}, \Delta_t F = \frac{-3F(t-\delta_t) + 4F(t) - F(t+\delta_t)}{2\delta_t}$$

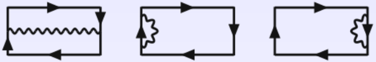
- We focus on the imaginary part which originates from Landau damping and is expected to exist in the classical limit

Results from the classical simulation



(Wilson loop dynamics and imaginary part of the potential)

Analytic expectations

 + ...

(Some diagrams contributing to the analytic result)

Analytic result (Laine et al., 2006)

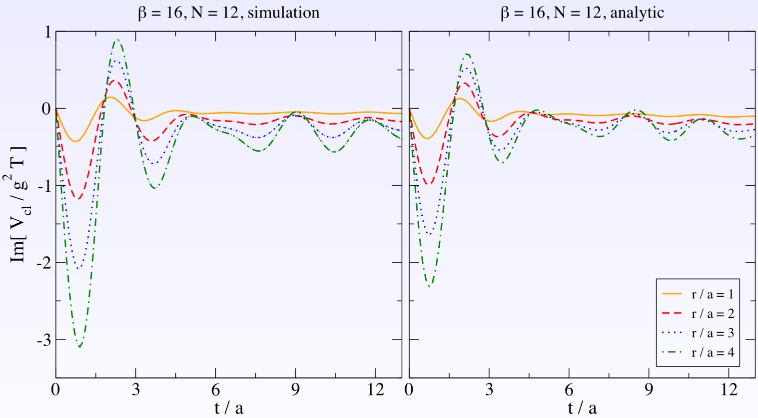
The Wilson Loop was also calculated using resummed perturbation theory followed by an analytic continuation leading to the following result for the real time static potential:

$$V_{>}^{(2)}(t,r) = g^2 C_F \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{2 - e^{iq_3 r} - e^{-iq_3 r}}{2} \left\{ \frac{1}{\mathbf{q}^2 + \Pi_E(0,\mathbf{q})} + \int_{-\infty}^{\infty} \frac{dq^0}{\pi} n_B(q^0) q^0 \right.$$

$$\left. \times \left(e^{\beta q^0} e^{-iq^0 t} - e^{iq^0 t} \right) \left[\left(\frac{1}{\mathbf{q}^2} - \frac{1}{(q^0)^2} \right) \rho_E(q^0,\mathbf{q}) + \left(\frac{1}{q_3^2} - \frac{1}{\mathbf{q}^2} \right) \rho_T(q^0,\mathbf{q}) \right] \right\}$$

These expressions have been evaluated using lattice regularization...

Comparison to the numerical results



...and compared to the numerical results.

Asymptotics: $t \to \infty$

	β_3	N	am_D	r=1a	r=2a	r=3a	r=4a
Simulation*	16.0	12	0.0	-0.060(2)	-0.156(8)	-0.246(26)	-0.319(56)
(~ 200 Config.)	16.0	16	0.0	-0.059(2)	-0.155(8)	-0.245(22)	-0.326(48)
	16.0	12	0.211	-0.059(2)	-0.147(7)	-0.229(23)	-0.297(51)
	16.0	12	0.350	-0.030(2)	-0.064(5)	-0.096(12)	-0.118(21)
	13.5	12	0.250	-0.071(2)	-0.174(10)	-0.270(33)	-0.341(97)
Analytic	16.0	∞	0.0	-0.0816	-0.1453	-0.1847	-0.2072

(Detailed overview of the results in the limit $t \to \infty$)

*The results for the simulation where obtained by fitting a constant to the region $t = 15a - 30a$.

Conclusion

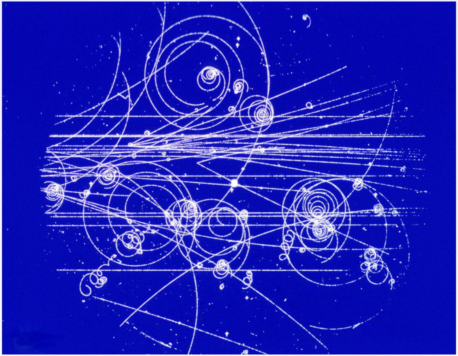
Result

The analytic result from resummed perturbation theory for the imaginary part of the real-time static potential has been confirmed using the drastically different approach of semiclassical lattice simulations.

Some open questions

- How to reproduce the quantum mechanical real part of the potential/ measure the spectral function directly on the lattice ?
- How to measure the real time potential of dynamical fermions ?

Questions



Questions?