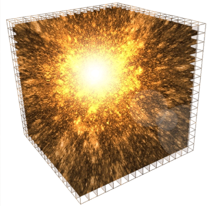

Real-Time Techniques for Thermal QCD

and their Application to Heavy Quark Physics

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Topics of this talk

- The talk will cover both analytical and numerical real-time techniques in two parts: Schwinger-Keldysh formalism and semi-classical lattice simulations.
- Analytical techniques are demonstrated by calculating the real-time static potential which generalizes the QCD static potential to a thermal setting. The physical signature of the $q\bar{q}$-resonance is discussed.
- Real-time lattice simulations are used to estimate non-perturbative corrections to the heavy quark diffusion constant and the static potential.

References

M. Laine, G. D. Moore, O. Philipsen and M. Tassler, *"Heavy Quark Thermalization in Classical Lattice Gauge Theory: Lessons for Strongly-Coupled QCD,"* JHEP 05(2009)14; M. Tassler, *"Heavy Quarkonia beyond Deconfinement and Real Time Lattice Simulations,"* arXiv:0812.3225 [hep-lat]; M. Laine, O. Philipsen and M. Tassler, *"Thermal imaginary part of a real-time static potential from classical lattice gauge theory simulations,"* JHEP 0709, 066; M. Laine, O. Philipsen, P. Romatschke and M. Tassler, *"Real-time static potential in hot QCD,"* JHEP 0703, 054;

Collaborators: M. Laine, G. D. Moore, O. Philipsen, P. Romatschke

In the following a general quantum statistical system characterized by a set $\{\varphi_i(x)\}, x \in \mathbb{R}^{d+1}$ of bosonic quantities $\varphi_i(x) \in \mathbb{R}$ is considered. The configuration space is the Hilbert space $\mathscr{H}_\varphi$.

Expectation Values

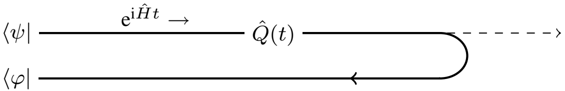
The expectation value of an operator $\hat{Q} : \mathscr{H}_\varphi \to \mathscr{H}_\varphi$ is defined via:

$$\langle \hat{Q} \rangle = \frac{1}{Z} \mathrm{Tr}\, \hat{\sigma} \hat{Q}, \qquad Z = \mathrm{Tr}\, \hat{\sigma}.$$

No restrictions are made on the nature of the *statistical operator* $\hat{\sigma}$ or the operator $\hat{Q}$ which may be time dependent and non-local.

$\hat{\sigma}$ won't necessarily commute with the Hamiltonian $\hat{H}$ of the system and may therefore be time dependent. Definition used: $\hat{\sigma} = \hat{\sigma}(t = 0)$.

Path Integral Representation



Weight: $\frac{1}{Z}\langle\varphi|\hat{\sigma}|\psi\rangle$

Complex time contour $\mathscr{C}$

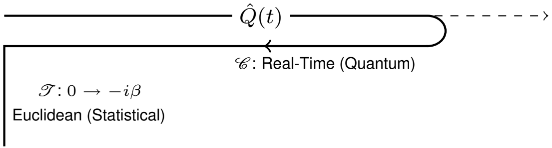
Path Integral and Weighting

$$\frac{1}{Z}\int[D\varphi]\ \langle\varphi|\hat{\sigma}\hat{Q}|\varphi\rangle \underset{\text{Insert 1}}{=} \frac{1}{Z}\int[D\varphi][D\psi]\langle\varphi|\hat{\sigma}|\psi\rangle|\langle\psi|\hat{Q}|\varphi\rangle$$

$$= \underbrace{\frac{1}{Z}\int[D\varphi][D\psi]\langle\varphi|\hat{\sigma}|\psi\rangle}_{\text{Statistical Weighting}}\,|\,\underbrace{\int_{\mathscr{C}}[D\phi(t>0)]\ \mathrm{e}^{\mathrm{i}S(\varphi)}Q(t)}_{\text{Quantum Evolution}}$$

The integral is build along a closed complex contour $\mathscr{C}:\mathbb{R}\to\mathbb{C}$, starting with the state $|\varphi\rangle$ and ending with state $|\psi\rangle$ of $\mathscr{H}_\varphi(t=0)$. The customary choice is the *Schwinger-Keldysch contour* depicted above.

Special Case: Thermal Equilibrium

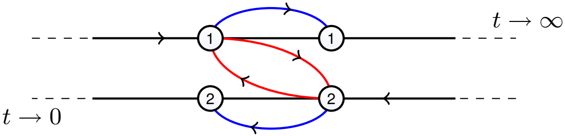


Thermal Equilibrium

$$\frac{1}{Z}\int [D\varphi]\ \langle\varphi|\hat{\sigma}|\varphi\rangle = \frac{1}{Z}\underbrace{\int [D\varphi][D\psi]\langle\varphi|\mathrm{e}^{-\beta\hat{H}}|\psi\rangle}_{\text{Statistical Weighting}} \mid \underbrace{\int_{\mathscr{C}} [D\phi(t)]\ \mathrm{e}^{\mathrm{i}S(\varphi)}Q(t)}_{\text{Quantum Evolution}}$$

$$= \frac{1}{Z}\underbrace{\int_{\mathscr{C}+\mathscr{T}} [D\phi(t)]\ \mathrm{e}^{\mathrm{i}S(\varphi)}Q(t)}_{\text{Finite Temperature Path Integral}}$$

In thermal equilibrium the statistical operator $\hat{\sigma} = \frac{1}{Z}\mathrm{e}^{-\beta\hat{H}}$ translates into a euclidean branch $\mathscr{T}$ of the time contour. For time independent quantities the real-time part $\mathscr{C}$ is omitted leading to a purely euclidean theory.



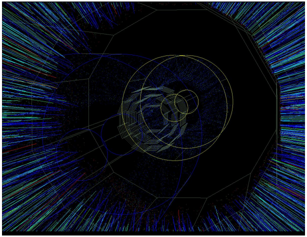
A location on $\mathscr{C}$ is specified by a time $t \in \mathbb{R}$ and an additional index $i \in 1, 2$ corresponding to a location on the forward or backward part of the contour. The correlator of two operators $\hat{\varphi}, \hat{\psi}$ takes a matrix form:

Real-Time Correlators

$$i\boldsymbol{G} = i \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} \langle \mathcal{T} \hat{\psi}(t')\hat{\varphi}(t) \rangle & -\langle \hat{\varphi}(t)\hat{\psi}(t') \rangle \\ \langle \hat{\psi}(t')\hat{\varphi}(t) \rangle & \langle \tilde{\mathcal{T}} \hat{\psi}(t')\hat{\varphi}(t) \rangle \end{pmatrix}$$

Retarded, advanced and symmetric correlators:

$$\boldsymbol{R}^{-1} \cdot \boldsymbol{G} \cdot \boldsymbol{R} = \begin{pmatrix} 0 & G_A \\ G_R & G_S \end{pmatrix} \quad \text{where} \quad \mathbf{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$



Heavy Ion Collision

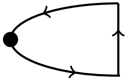
In the following this formalism will be employed to investigate the dynamics of the quark-gluon plasma.

Partition Function

In thermal equilibrium a purely gluonic system is characterized by the partition function

$$Z = \int_{\mathscr{C}+\mathscr{T}} [DA] \mathrm{e}^{iS}, \qquad S = -\frac{1}{4} Tr \int_{\mathscr{C}+\mathscr{T}} ds d^3x F_{\mu\nu} F^{\mu\nu}$$

where S is the Yang-Mills action along the contour $\mathscr{C} + \mathscr{T}$.

$$\mathrm{i}C_{21}(t,\boldsymbol{r}) =$$ 

Propagator for a $q\bar{q}$-pair

In the following this formalism will be employed to investigate the propagation of a $q\bar{q}$-pair with constituent mass $M \ll T$ in a quark-gluon plasma.

Static Potential at $T = 0$

At $T = 0$ the propagation of a heavy $q\bar{q}$ pair is described by a Schrödinger equation:

$$\mathrm{i}\partial_t C_{21}(t,\boldsymbol{r}) = \left(2M - \frac{\triangle_{\boldsymbol{r}}}{M} + V(t,\boldsymbol{r})\right) C_{21}(t,\boldsymbol{r}).$$

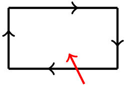
The potential $V(t,\boldsymbol{r})$ is approximated by the potential of a static $q\bar{q}$-pair:

$$V(t,\boldsymbol{r}) \simeq -C_F \frac{\alpha_S}{r} - \sigma r.$$

α_S: Coupling for single gluon exchange, σ: String Tension

At finite temperature it is unclear how to define a potential from first principles.

Real Time Static Potential



$$[i\partial_t - V(t,r)]\, C_{21}(t,r) = 0$$

Static Potential (Laine,Philipsen,Romatschke,Tassler,JHEP 0703 (2007) 054)

A static potential is defined in the large time and static mass limit of the Schrödinger equation:

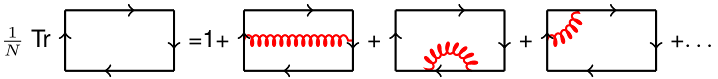
$$V(\mathbf{r}) = \lim_{t\to\infty} V(t,\mathbf{r})$$

The quarkonium correlator is subsequently calculated by solving the Schrödinger equation for physical quark masses:

$$\left(i\partial_t - [-\frac{\triangle_{\mathbf{r}}}{M} + V(\mathbf{r}) + 2M]\right) C_{21} = 0, \quad \text{BC: } C_{21}(t=0) \sim \delta(\mathbf{r})$$

Recent uses (2008,2009): A. Dumitru, Y. Guo and M. Strickland, *"The imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma,"* arXiv:0903.4703 [hep-ph]; N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky, *"Static quark-antiquark pairs at finite temperature,"* Phys. Rev. D **78** (2008); A. Beraudo, J. P. Blaizot and C. Ratti, *"Real and imaginary-time $Q\bar{Q}$ correlators in a thermal medium,"* Nucl. Phys. A **806** (2008) ; M. A. Escobedo and J. Soto, *"Non-relativistic bound states at finite temperature (I): the hydrogen atom,"* arXiv:0804.069 ; Y. Burnier, M. Laine and M. Vepsalainen, *"Heavy quarkonium in any channel in resummed hot QCD,"* JHEP **0801** (2008)

Expansion of the Wilson Loop



Diagrams contributing to the Wilson-Loop

The Real-Time static potential to $\mathcal{O}(g^2)$

$$V(\boldsymbol{r}) = g^2 \mathrm{C_F} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} (1 - \cos \boldsymbol{k} \cdot \boldsymbol{r}) \tilde{G}_{11}^{00}(\omega = 0, \boldsymbol{k}).$$

Here $\tilde{G}_{11}^{00}$ is the longitudinal component of the time ordered gluon propagator which can be decomposed as:

$$\tilde{G}_{11} = Re\, \tilde{G}_R + \frac{1}{2} \tilde{G}_S.$$

In the special case of thermal equilibrium the propagator is given by:

$$\tilde{G}_{11}^{00}(\omega \to 0, \vec{k}) = \underbrace{\frac{1}{\vec{k}^2 + m_D^2}}_{Re(G),\text{Retarded}} + \underbrace{i\frac{\pi m_D^2}{\beta k} \frac{1}{(\vec{k}^2 + m_D^2)^2}}_{\text{Im(G),Symmetric}}.$$

$m_D \simeq gT$ is the thermal gluon mass.

The static potential in thermal equilibrium

Plugging in the propagator the following result is obtained:

The static potential

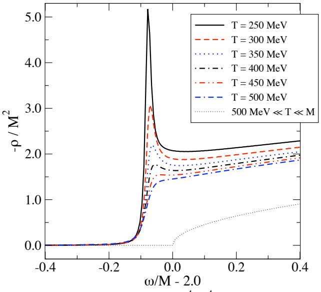
$$V(r) = \underbrace{-\frac{g^2 C_F}{4\pi}[m_D + \frac{exp(-m_D r)}{r}]}_{\text{Re(V): Retarded contribution}} \underbrace{-i\frac{g^2 T C_F}{2\pi}\phi(m_D r)}_{\text{Im(V): Symmetric contribution}}$$

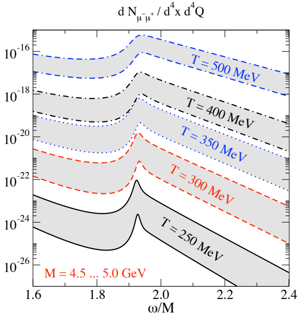
with $\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[1 - \frac{sin(zx)}{zx}\right]$

Features:

- The real part is the usual Debye screended potential (Contribution of the retarded propagator)
- An imaginary part appears at finite T due to Landau damping (Contribution of the symmetric propagator).

Physical signatures





Quarkonium signatures from the finite mass Schrödinger equation:

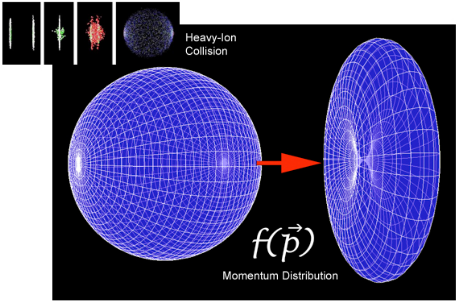
Spectral function [arXiv:0711.1743]

- The spectral function is depicted for Bottomonium.
- The imaginary part induces a finite width to the resonance peak (melting of the resonance).

Potential [arXiv:0704.1720]

- The Dilepton rate is shown for Bottomonium.
- A softening of the resonance is seen for increased temperature.

Anisotropic Media



Results have also been obtained for an anisotropic plasma characterized by the momentum distribution:

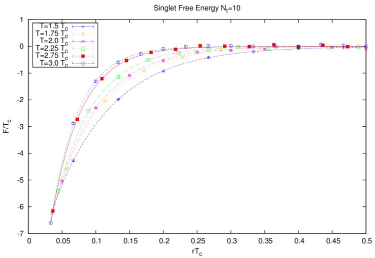
$$f(\vec{p}) = N(\xi) n_B(p\sqrt{1+\xi(\vec{v}_p \cdot \vec{n})^2})$$

n_B: Thermal Bose Distribution, ξ: Anisotropy, $\vec{n}$: Collision axis, $N(\xi) = \sqrt{1+\xi}$: Normalization

Both limiting cases $\xi \to \infty$ and $\xi \ll 1$ have been considered.

See also: A. Dumitru, Y. Guo and M. Strickland, *"The imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma,"* arXiv:0903.4703 [hep-ph]; Y. Burnier, M. Laine and M. Vepsalainen, *"Quarkonium dissociation in the presence of a small momentum space anisotropy,"* arXiv:0903.3467 [hep-ph].

Putting it on the Lattice ?



Singlet part of the free energy in lattice gauge theory (Wilson action, $N_C = 3$, $N_\tau = 10$)

Challenge

The Real-Time static potential is a dynamical quantity defined from a Minkowski space Schrödinger equation.

- The real and imaginary parts mix upon analytic continuation. Binding energy and decay width can not be seperated on the lattice.
- For $t = -i\beta$ the singlet free energy is recovered. This quantity can not be related to the $t \to \infty$ limit of the potential in Minkowski space.

Solution: Measure the correlator in (semi-)classical real-time simulations.

Classical Limit in Thermal Field Theory

Consider a general bosonic system characterized by a set of fields and momenta $\{\varphi_i(x), \pi_i(x)\}$. The parition function is:

$$Z = Tre^{-\beta \hat{H}(\{\hat{\phi}_i, \hat{\pi}_i\})}$$

Thermal insertions in perturbation theory (bare symmetric propagators) are proportional to the momentum distribution:

$$n_B(\omega = k) + \frac{1}{2} = \frac{T}{\hbar\omega} + \frac{1}{12}\frac{\hbar\omega}{T} + \ldots$$

Since the expansion parameter is $\hbar g^2$ the classical limit $\hbar \to 0$ contains all diagrams with $2m$ vertices and a maximal number of m thermal insertions at soft momenta $\omega_j \ll T, j \in \{1, \ldots, m\}$:

$$g^{2m} \prod_{j=1}^{m} \left(\frac{T}{\omega_j}\right), \quad m \in \mathbb{N} \qquad \textit{(Classical Contributions)}.$$

For theories where soft momentum scales can be separated from hard scales these diagrams will dominate the low energy region $\omega \ll T$.

The Heisenberg picture equations of motion for fields and momenta are:

$$\frac{d}{dt}\hat{\varphi} = \frac{i}{\hbar}[\hat{H}, \hat{\varphi}] \quad \text{and} \quad \frac{d}{dt}\hat{\pi} = \frac{i}{\hbar}[\hat{H}, \hat{\pi}].$$

Using the bosonic commutation relations

$$[\hat{\phi}, \hat{\pi}] = i\hbar$$

the commutators are identified as derivatives in the classical limit $\hbar \to 0$:

$$\lim_{\hbar \to \infty} \frac{i}{\hbar}[\hat{\pi}^n, \hat{\varphi}] = -n\hat{\pi}^{n-1} \quad \text{and} \quad \lim_{\hbar \to \infty} \frac{i}{\hbar}[\hat{\varphi}, \hat{\pi}] = n\hat{\varphi}^{n-1}$$

By writing the Hamiltonian $\hat{H}(\{\varphi_i(x), \psi_i(x)\})$ as a power series the classical Hamiltonian equations of motion are recovered:

$$\frac{d}{dt}\hat{\varphi}_i(x) = \frac{\delta \hat{H}}{\delta \hat{\pi}_i(x)} \quad \text{and} \quad \frac{d}{dt}\hat{\pi}_i(x) = -\frac{\delta \hat{H}}{\delta \hat{\varphi}_i(x)}$$

Application to the Yang-Mills Plasma

Momentum Scales of the Yang-Mills Plasma:

- *Hard Momentum Scale:* $k \sim T$

  The characteristic gluon momentum scale corresponds to the temperature T. The length scale $1/T$ is the average inter particle spacing.

- *Electric Scale:* $k \sim gT$

  The thermal mass of longitudinal gluons is $m_D \sim gT$. This translates into a supression of electrical interactions at distances $> 1/m_D$.

- *Magnetic Scale:* $k \sim g^2 T$

  Chromo-magnetism is dynamically screened over distances larger than $1/g^2 T$. Interactions become non-perturbative.

Problem: Soft and hard scales are not seperated. Hard loop contributions dominate in soft gluon exchange. A systematic expansion in $\hbar$ is necessary.

Classical Limit on the Lattice

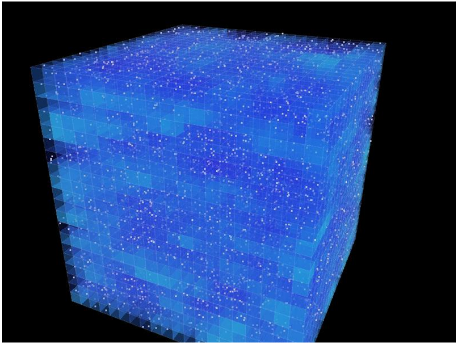


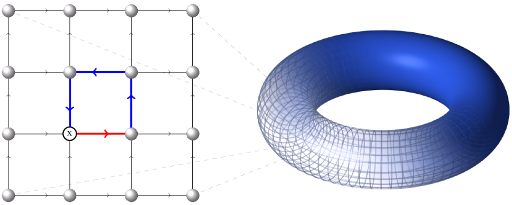
Illustration of a lattice configuration (Includes classical particles).

The simplest way to study the classical limit of thermal Yang-Mills theory on a lattice is to isolate soft fields by restricting spatial momenta to a Brillouin zone including only regions of momentum space with high occupation numbers:

$$k_i < \Lambda_{cl}, \qquad gT < \Lambda_{cl} < T$$

Here the classical limit $\hbar \to 0$ can safely be taken. Observables must be insensitive to physics at the hard scale.

Hamiltonian Lattice



Spatial gauge fields are discretized on a lattice in the usual way. The (Minkowskian) time is continuous. Red: Link $U_i(x)$, Blue: Plaquette $U_{ij}(x)$.

The Hamiltonian lattice:

- Discretization: $\mathbb{R}^{d+1} \to \mathbb{Z}^d \times \mathbb{R}^1$. To apply classical statistical mechanics in a straightforward way a temporal gauge is chosen.
- Magnetic Fields: Spatial gauge fields are discretized as links

$$W(x, x+\hat{i}) \to U_i(x)$$

  where W is a straight Wilson line connecting the sites x and $x+\hat{i}$.
- Electric Fields: Electric fields are defined via the relation:

$$\dot{A}_i = E_i \to \dot{U}_i = iE_iU_i$$

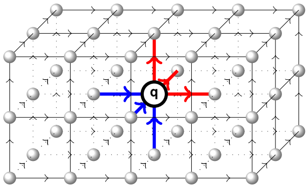


Illustration of Gauss's law $G(E,U)=q$

The partition function of the classical lattice model is

Classical Partition Function

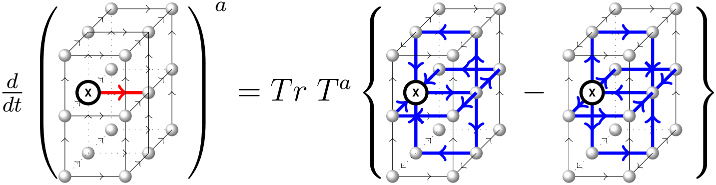
$$Z_L = \int [DU] \int [DE] \delta(G) e^{-\beta H_L}, \quad H_L(t) = \sum_x TrE_iE_i + H_M$$

where the magnetic part H_M is identical to a d-dimensional Wilson action:

$$H_M = \beta_L \sum_x \left\{ 1 - \frac{1}{N} \sum_{i<j} ReTrU_{ij}(x,t) \right\}.$$

Color electric fields can also be integrated out to arrive at the partition function for a Yang-Mills theory coupled to an adjoint massless Higgs $A_0^a(x)$.

Equations of Motion



Ampere's law on the lattice. Red: electric fields, Blue: spatial gauge fields.

The Euler-Lagrange equations are obtained from a variation of the lattice action with respect to the entries of link variables under a unitarity constraint.

Euler-Lagrange Equations

$$\begin{aligned} \dot{U}_i(x,t) &= iE_i(x,t)U_i(x,t) && \textit{(Faraday's law of induction)} \\ \dot{E}_i^a(x,t) &= 2\sum_{|j|\neq i} ImTr\left\{T^a U_{ij}(x,t)\right\} && \textit{(Ampere's circuital law)} \end{aligned}$$

All ensemble configurations satisfy the Gauss constraint $G = 0$:

$$\sum_i \left\{E_i(x,t) - U_{-i}(x,t)E_i(x-\hat{\imath},t)U_{-i}^{+}(x,t)\right\} = 0 \qquad \textit{(Gauss's law).}$$

The constraint remains satisfied during the time evolution for individual ensemble configurations.

Matching to the Quantum Theory

The theory can be matched to quantum theory in the continuum by restricting momenta to the Brillouin zone Λ_{cl} in resummed perturbation theory. The following hirarchy of length scales appears:

1. *Hard Scale* a

   Most energy resides at the cutoff scale $\sim \pi/a$ where the thermal energy distribution is cut off artificially.

2. *Electric Scale* $\sqrt{a/g^2T}$

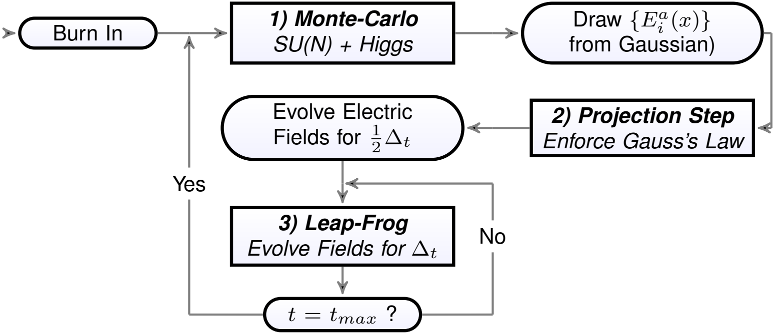
   Beyond this length scale electrical interactions are screened due to the longitudinal gluon mass $m_{D,L}$.

3. *Magnetic Scale* π/g^2T

   Magnetic interactions, which remain the most far reaching interactions in the lattice model, are screened dynamically at this length scale.

The physics at soft scales is similar to the continuum while discretization effects dominate the hard scale. Lattice hydrodynamics is badly affected by the maximal deformation of momenta at the hard scale.

Lattice algorithm



1. Link variables are generated by an overrelaxed heatbath for gauge fields with an adjoint Higgs. This procedure essentially eliminates thermalization times (up to 60% of simulation time in older algorithms).
2. Electrical fields are drawn from a Gaussian and projected on the hypersurface proscribed by the gauge fixing.
3. A leapfrog is chosen as symplectic integrator for the Euler Lagrange equations. The initial step is the only source of $\mathscr{O}(a)$ errors.

Implementation Status

	SU(2)	SU(3)
Purely Classical		
Standard action	$\surd$	$\surd$
Improved action		
Hard Thermal Loop		
Spherical Harmonics	$\surd$	$\surd$
Non-Equilibrium	$\surd$	$\surd$
Romatschke's Discretization	$\surd$	$\surd$
Wong Equations		
Particles	$\surd$	$\surd$

$\surd$: Implemented and checked in collaboration, $\surd$: Implemented, $\surd$: Testing

The number of spatial dimensions can be varied where applicable. Parallel implementation via QDP++ available.

Application 1: Heavy-Quark Diffusion

The random walk of a heavy quark of mass $M \gg T$ in a quark gluon plasma is described by a Langevin equation:

Langevin equation

$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t), \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa \delta_{ij}\delta(t-t')$$

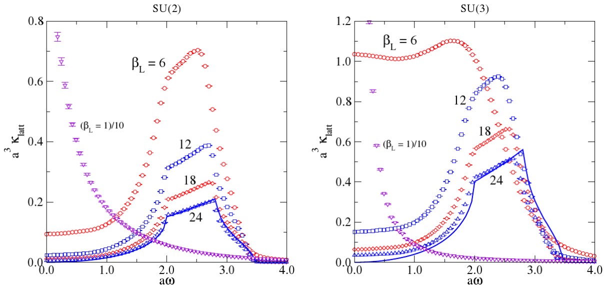
- The momentum diffusion constant κ and relaxation rate η_D are related by a fluctution dissipation relation:

$$\eta_D = \frac{\kappa}{2MT}$$

- The coefficient $\kappa = \kappa(0)$ is obtained form the electric field correlator

$$\kappa(\omega) = \frac{g^2}{3N} \int_{-\infty}^{\infty} dt e^{i\omega t} Tr\langle W^+(t,0)E_i(t,x)W(t,0)E_i(0,x)\rangle$$

with a Wilson line $W(t,0)$ connecting the chromo electric fields.



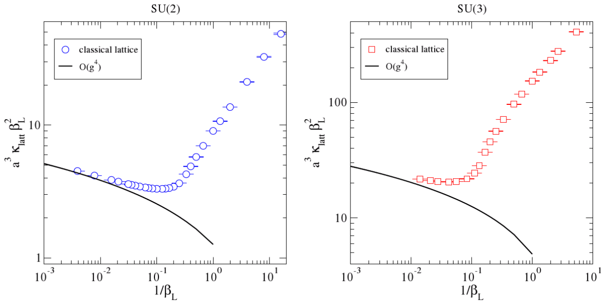
Fourier transform of $\kappa(t)$. Solid line: perturbation theory ($\beta_L = 24$)

Obervable: E-Field Correlator

$$\kappa(t) = \frac{1}{3N} Tr\langle E_i(t')E_i(t'+t)\rangle$$

- The shape of the correlator agrees with perturbative predictions. The peak structure at high ω reflects the introduction of a Brillouin zone.
- The correlator is flat at $\omega = 0$. The diffusion constant can potentially be measured in euclidean lattice simulations[Caron-Huot,Laine,Moore:0901.1195]

Lessons for Strong Coupling



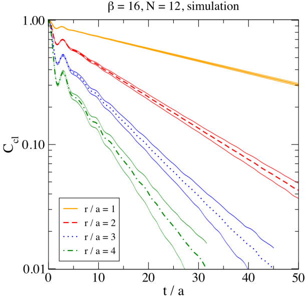
Heavy quark diffusion constant $\kappa = \kappa(\omega = 0)$. Solid line: perturbation theory

Leading order perturbative result (Moore,Teaney,Phys.Rev.C71 (2004))

$$\kappa \simeq \frac{g^2 C_F T m_D^2}{6\pi} \left(\frac{T}{m_D} + \dots \right)$$

- Non-perturbative corrections in the classical theory are large at physical temperatures.
- In the strong coupling regime $\beta_L^{-1} = a(g^2 T/2N) \to \infty$ lattice artifacts dominate and results depend on the discretization scheme.

Application 2: Real-Time Static Potential



Time evolution of the Wilson loop $C_{11}(t,r)$. Outer lines: Jacknife tolerance

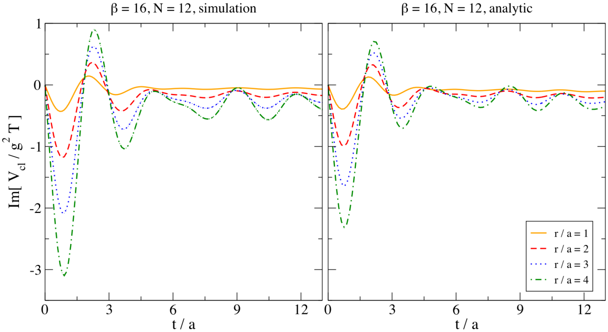
Obervable: Wilson Loop

$$C_{11}(t,r) = \frac{1}{N} Tr\langle W(t',r) W^{+}(t'+t,r)\rangle$$

The time evolution of a Wilson Loop of spatial extent r was measured. The quarkonium potential is obtained via:

$$V(t,r) = \frac{i\partial_t C_{11}(t,r)}{C_{11}(t,r)}$$

Validity of the Classical Approximation



	β_3	N	am_D	r=1a	r=2a	r=3a	r=4a
Simulation* (∼ 200 Config.)	16.0	12	0.0	-0.060(2)	-0.156(8)	-0.246(26)	-0.319(56)
	16.0	16	0.0	-0.059(2)	-0.155(8)	-0.245(22)	-0.326(48)
	16.0	12	0.211	-0.059(2)	-0.147(7)	-0.229(23)	-0.297(51)
	16.0	12	0.350	-0.030(2)	-0.064(5)	-0.096(12)	-0.118(21)
	13.5	12	0.250	-0.071(2)	-0.174(10)	-0.270(33)	-0.341(97)
Analytic	16.0	∞	0.0	-0.0816	-0.1453	-0.1847	-0.2072

(Overview of the asymptotic results for $t \longrightarrow \infty$) (Laine, Philipsen, Tassler, arXiv,0707 2458)

Summary

- Expansions:
  The Schwinger-Keldysh formalism (expansion in g^2) and the classical approximation (expansion in $\hbar$) were highlighted.
- Quantities:
  The static $q\bar{q}$-potential in a thermal medium and quarkonium resonance were discussed. Non-perturbative corrections to the heavy quark diffusion constant due to soft classical physics were estimated on the lattice.
- Challenge:
  A quantitative understanding of nonperturbative effects beyond leading order or kinetic theory is indispensible to connect experiment and theory.